Modal Interactions in an Autoparametric Vibration Absorber to Narrow Band Random Excitation

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The main objectives of this study are to examine the random responses of a vibration absorber system with autoparametric coupling in the neighborhood of internal resonance subjected to narrow band random excitation by Gaussian closure scheme and to compare the results with those obtained by Monte Carlo simulation. The Monte Carlo simulation is found to support the main features of the nonlinear modal interaction in the neighborhood of internal resonance conditions. The jump phenomenon of the cantilever mode and saturation phenomenon of the main system are shown to occur if the excitation bandwidth is sufficiently small.

Key Words : Internal Resonance, Autoparametric Vibration Absorber, Narrow Band Random Excitation, Gaussian Closure Scheme, Jump, Saturation Phenomenon, Monte Carlo Simulation

1. Introduction

The linear modeling of any dynamical system is commonly accepted as long as the actual response characteristics to various types of loading show general agreement with follow the linear solution. However, under certain situations the system may experience certain complex characteristics that cannot be justified by the linear solution. These complex response features owe their origin to inherent nonlinearities in the system. When natural frequencies, ω_i, of a non-linear multi-degree-of-freedom system are commensurable or nearly so, i.e., when they satisfy the internal resonance condition $\sum\lambda_i\omega_i=0$, where λ_i are integers, the system may possess internal resonances (modal interactions). Under this condition, the response of the system can exhibit modal interaction in the form of an energy exchange through non-linear coupling between normal modes of the system. Modal interactions of harmonically excited non-linear systems with internal resonance have been studied extensively by Minorsky (1962), Haxton and Barr (1972), Nayfeh and Mook (1979), and Lee and Hsu (1994). These systems have been known to exhibit complicated behaviors such as jump and saturation phenomena, Hopf bifurcations and a sequence of period-doubling bifurcations leading to chaos.

Recent developments in the theory of stochastic processes and stochastic differential equations have been restricted to handling limited classes of dynamical systems. The theory has provided answers to a number of issues, including stochastic stability, on-off intermittency, chaos, and bifurcation of multiplicative noise. However, there can be no general rule about the suitability of a given method for a particular nonlinear system. Furthermore, the application of different techniques to the same system may lead to different results. Nevertheless modal interactions of non-linear multi-degree-of-freedom systems with internal resonance subjected to random excitation have been studied by many authors. For example, Ibrahim and Roberts (1976, 1977), and Roberts (1980) included cubic non-linear terms in the

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analysis for systems with 1 : 2 internal resonance subjected to broadband random excitation, and the destabilizing effect of a damping ratio was observed. Later, Lee and Cho (2000) could not find the destabilizing effect of damping, which was observed by Ibrahim and Roberts (1977). Ibrahim and Li (1988) employed closure schemes to examine the response statistics of a nonlinear three degree-of-freedom system under wide band random excitation and found regions of multiple solutions in the neighborhood of exact internal resonance, but the numerical solution (Li and Ibrahim, 1990) yields only one solution depending on the assigned initial condition. Cho and Lee (2000) examined the response statistics of a continuous system under wide band random excitation and showed that there exists no significant difference between two- and three-mode interactions. Ibrahim (1991) and Lee and Cho (1998) could not find saturation phenomenon for dynamical systems with quadratic nonlinearity subject to wide band random excitation.

Many of the phenomena observed in the deterministic system, such as jump and saturation, do not appear in the case of the white noise excitation as stated in the above studies. The motive of this study arises from the conjecture that these phenomena may be observable as the bandwidth of the excitation is sufficiently decreased. Accordingly we select an autoparametric vibration absorber subjected to a narrow band random excitation in order to investigate the influence of the internal resonance. The narrow band random process can be generated from a linear shaping filter excited by a white noise. Obtaining moment equations from the Fokker-Planck equation corresponding to the coupled non-linear ordinary differential equations, we use Gaussian and non-Gaussian closure schemes to reduce a system of autonomous ordinary differential equations for moments but can not get any solution by the non-Gaussian closure scheme because the solution experiences divergence. The response statistics by Gaussian closure is examined. The results obtained by Gaussian closure scheme are compared with those obtained by Monte Carlo simulation.

2. Equations of Motion

Figure 1 shows the autoparametric system under narrow band random excitation $F(t)$. The equations of motion of the system (Haxton and Barr, 1972) are, for the main mass,

$$\begin{aligned}&(M+m)\ddot{x}^*+c_1\dot{x}^*+k_1x^*\\&-(6/5l)\,m(\dot{y}^2+y\ddot{y})=F(t)\end{aligned}\tag{1}$$

and, for the cantilever,

$$\begin{aligned}&m\ddot{y}+c_2\dot{y}+\{k_2-(6/5l)\,m\ddot{x}^*\}y\\&+(36/25l^2)\,my(\dot{y}^2+y\ddot{y})=0\end{aligned}\tag{2}$$

where x^* and y are normal coordinates corresponding to the linearized system. Introducing the notations

$$\begin{aligned}&X=\frac{x^*}{x_s^*},\ Y=\frac{y}{l},\ \zeta_1=\frac{c_1}{2(M+m)\omega_1},\ \zeta_2=\frac{c_2}{2m\omega_2},\\&\tau=\omega_1 t,\ r=\frac{\omega_2}{\omega_1},\ \varepsilon=\frac{l}{x_s^*},\ R=\frac{m}{M+m},\\&\omega_1^2=\frac{k_1}{M+m},\ \omega_2^2=\frac{k_2}{m},\ U(\tau)=\frac{F(\tau/\omega_1)}{(M+m)x_s^*\omega_1^2},\ \rho=\frac{6}{5}\end{aligned}\tag{3}$$

we have the nondimensionalized equations as follows :

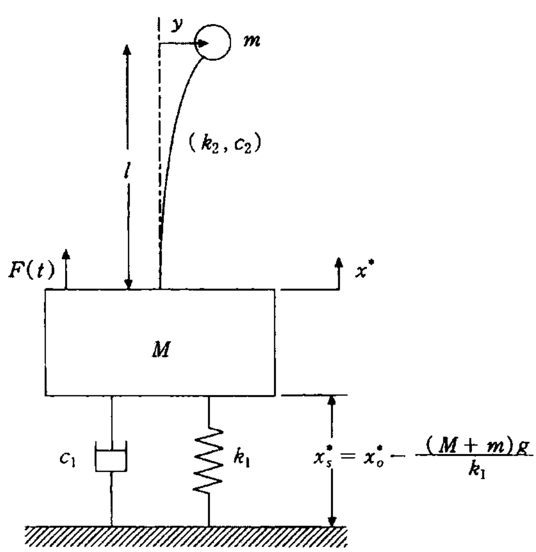

Fig. 1 Schematic diagram of an autoparametric absorber system

$$X''+2\zeta_1X'+X-\rho\varepsilon R(Y'^2+YY'')=U(\tau)$$
$$Y''+2\zeta_2 rY'+\left(r^2-\frac{\rho}{\varepsilon}X''\right)Y+\rho^2Y(Y'^2+YY'')=0 \quad (4a)$$

where $U(\tau)$ is the response of the linear filter equation

$$U''+\Psi U'+\Omega^2U=W(\tau) \quad (4b)$$

with center frequency Ω and bandwidth Ψ, and dot and prime denote differentiations with respect to t and τ, respectively. Random excitation $W(\tau)$ is assumed to be zero mean white noise having the autocorrelation function

$$R_{WW}(\Delta\tau)=E[W(\tau)W(\tau+\Delta\tau)]=2D\delta(\Delta\tau) \quad (5)$$

where $2D$ represents the spectral density when we express the frequency by $f(=\Omega/2\pi)$, and $\delta(\Delta\tau)$ is the Dirac delta function.

Eliminating the nonlinear acceleration terms and neglecting the fourth and higher orders of nonlinear terms we have

$$X''+2\zeta_1X'+X+\rho\varepsilon R(-Y'^2+2r\zeta_2YY'+r^2Y^2)+\rho^2R(-U(\tau)Y^2+2\zeta_1X'Y^2+XY^2)=U(\tau),$$
$$Y''+2\zeta_2rY'+r^2Y+\frac{\rho}{\varepsilon}(-YU(\tau)+2\zeta_1YX'+XY)+\rho^2(1-R)(-r^2Y^3-2\zeta_2rY^2Y'+YY'^2)=0, \quad (6)$$
$$U''+\Psi U'+\Omega^2U=W(\tau)$$

3. Moment Equations By Closure Schemes

Introducing the notations

$$\{X,\ Y,\ X',\ Y',\ U,\ U'\}^T=\{X_1,\ X_2,\ X_3,\ X_4,\ X_5,\ X_6\}^T=\boldsymbol{X}$$

and letting $W(\tau)$ be a formal derivative of a Brownian process, i.e., $W(\tau)=dB(\tau)/d\tau$, we can express Eq. (6) in the form of the Itô stochastic equation :

$$dX_1=X_3d\tau,\ dX_2=X_4d\tau,$$
$$dX_3=\{-2\zeta_1X_3-X_1+X_5+\rho\varepsilon R(X_4^2-2r\zeta_2X_2X_4-r^2X_2^2)+\rho^2R(X_2^2X_5-2\zeta_1X_2^2X_3-X_1X_2^2)\}d\tau,$$
$$dX_4=\left\{-2\zeta_2rX_4-r^2X_2+\frac{\rho}{\varepsilon}(X_2X_5-2\zeta_1X_2X_3-X_1X_2)+\rho^2(1-R)(r^2X_2^3+2\zeta_2rX_2^2X_4-X_2X_4^2)\right\}d\tau, \quad (7)$$
$$dX_5=X_6d\tau,$$
$$dX_6=(-\Psi X_6-\Omega^2X_5)d\tau+dB(\tau)$$

The solution process of this equation is a Markov process and the Fokker-Planck equation may be applied for the Markov vector $\boldsymbol{X}$ in the form

$$\frac{\partial}{\partial\tau}p(\mathbf{x},\ \tau)=-\sum_{i=1}^{6}\frac{\partial}{\partial x_i}[a_i(\mathbf{x},\ \tau)p(\mathbf{x},\ \tau)]+\frac{1}{2}\sum_{i=1}^{6}\sum_{j=1}^{6}\frac{\partial^2}{\partial x_i\partial x_j}[b_{ij}(\mathbf{x},\ \tau)p(\mathbf{x},\ \tau)] \quad (8)$$

where $p(\mathbf{x},\ \tau)$ is the joint probability density function, and $a_i(\mathbf{x},\ \tau)$ and $b_{ij}(\mathbf{x},\ \tau)$ are the first and second incremental moments of the Markov process $\boldsymbol{X}(\tau)$. These are defined as follows :

$$a_i(\mathbf{x},\ \tau)=\lim_{\delta\tau\to0}\frac{1}{\delta\tau}E\{X_i(\tau+\delta\tau)-X_i(\tau)\,|\,\boldsymbol{X}(\tau)=\mathbf{x}\},$$
$$b_{ij}(\mathbf{x},\ \tau)=\lim_{\delta\tau\to0}\frac{1}{\delta\tau}E\{[X_i(\tau+\delta\tau)-X_i(\tau)][X_j(\tau+\delta\tau)-X_j(\tau)]\,|\,\boldsymbol{X}(\tau)=\mathbf{x}\} \quad (9)$$

From Eq. (7) a_i and b_{ij} are evaluated as follows :

$$a_1=x_3,\ a_2=x_4,$$
$$a_3=-2\zeta_1x_3-x_1+x_5+\rho\varepsilon R(x_4^2-2r\zeta_2x_2x_4-r^2x_2^2)+\rho^2R(x_2^2x_5-2\zeta_1x_2^2x_3-x_1x_2^2),$$
$$a_4=-2\zeta_2rx_4-r^2x_2+\frac{\rho}{\varepsilon}(x_2x_5-x_1x_2-2\zeta_1x_2x_3)+\rho^2(1-R)(r^2x_2^3+2\zeta_2rx_2^2x_4-x_2x_4^2), \quad (10)$$
$$a_5=x_6,\ a_6=-\Psi x_6-\Omega^2x_5,$$
$$b_{66}=2D,\ \text{all other } b_{ij}=0$$

Since it is impossible to obtain the exact solution $p(\mathbf{x},\ \tau)$ to the Fokker-Planck equation, we examine the system responses by means of moment equations. First, introducing the following notations for nth-order moments of the system responses,

$$m_{\alpha,\beta,\gamma,\eta,\iota,\nu}(\tau)=E[X_1^\alpha X_2^\beta X_3^\gamma X_4^\eta X_5^\iota X_6^\nu]=\iiiint_{-\infty}^{\infty}x_1^\alpha x_2^\beta x_3^\gamma x_4^\eta x_5^\iota x_6^\nu p(\mathbf{x},\ \tau)\,dx_1dx_2dx_3dx_4dx_5dx_6$$

with $n(=\alpha+\beta+\gamma+\eta+\iota+\nu)$, we can derive a set of dynamic moment equations of any order by multiplying Eq. (10) by $x_1^\alpha x_2^\beta x_3^\gamma x_4^\eta x_5^\iota x_6^\nu$ and integrating by parts over the entire state space $-\infty < x_i < \infty$. This procedure results in the following general dynamic moment equation:

$$
\begin{aligned}
m_{\alpha,\beta,\gamma,\eta,\iota,\nu} = {} & \alpha m_{\alpha-1,\beta,\gamma+1,\eta,\iota,\nu} + \beta m_{\alpha,\beta-1,\gamma,\eta+1,\iota,\nu} - \gamma m_{\alpha+1,\beta,\gamma-1,\eta,\iota,\nu} \\
& - \varepsilon\rho R r^2 \gamma m_{\alpha,\beta+2,\gamma-1,\eta,\iota,\nu} - \rho^2 R \gamma m_{\alpha+1,\beta+2,\gamma-1,\eta,\iota,\nu} \\
& + \varepsilon\rho R \gamma m_{\alpha,\beta,\gamma-1,\eta+2,\iota,\nu} + \gamma m_{\alpha,\beta,\gamma-1,\eta,\iota+1,\nu} \\
& + \rho^2 R \gamma m_{\alpha,\beta+2,\gamma-1,\eta,\iota+1,\nu} - 2\zeta_1 \gamma m_{\alpha,\beta,\gamma,\eta,\iota,\nu} \\
& - 2\rho^2 R \zeta_1 \gamma m_{\alpha,\beta+2,\gamma,\eta,\iota,\nu} - 2\varepsilon\rho R r \zeta_2 \gamma m_{\alpha,\beta+1,\gamma-1,\eta+1,\iota,\nu} \\
& - r^2 \eta m_{\alpha,\beta+1,\gamma,\eta-1,\iota,\nu} - \frac{\rho}{\varepsilon} \eta m_{\alpha+1,\beta+1,\gamma,\eta-1,\iota,\nu} \\
& + \rho^2 (1-R) r^2 \eta m_{\alpha,\beta+3,\gamma,\eta-1,\iota,\nu} \\
& - \rho^2 (1-R) \eta m_{\alpha,\beta+1,\gamma,\eta+1,\iota,\nu} \\
& + \frac{\rho}{\varepsilon} \eta m_{\alpha,\beta+1,\gamma,\eta-1,\iota+1,\nu} - 2\frac{\rho}{\varepsilon} \zeta_1 \eta m_{\alpha,\beta+1,\gamma+1,\eta-1,\iota,\nu} \\
& - 2 r \zeta_2 \eta m_{\alpha,\beta,\gamma,\eta,\iota,\nu} + 2\rho^2 (1-R) r \eta m_{\alpha,\beta+2,\gamma,\eta,\iota,\nu} \\
& + \iota m_{\alpha,\beta,\gamma,\eta,\iota-1,\nu+1} - \Omega^2 \nu m_{\alpha,\beta,\gamma,\eta,\iota+1,\nu-1} \\
& - \Psi \nu m_{\alpha,\beta,\gamma,\eta,\iota,\nu} + D\nu(\nu-1) m_{\alpha,\beta,\gamma,\eta,\iota,\nu-2}
\end{aligned}
\tag{11}
$$

Equation (11) constitutes a set of infinite coupled equations. In other words, the differential equation of order n contains moment terms of order $n+1$ and $n+2$. The Gaussian closure scheme is based on the assumption that the response process is nearly Gaussian and is carried out by setting third- and fourth-order cumulants to zero. The third- and fourth-order moments can be expressed in terms of lower-order moments (Lin and Cai, 1995). For non-Gaussian processes the cumulants of order higher than the second do not vanish. However, their contribution diminishes as their order increases if the process deviates slightly from Gaussian. Thus, non-Gaussian closure is carried out by setting fifth- and sixth-order cumulants to zero and expressing fifth- and sixth-order moments in terms of lower order moments (Lin and Cai, 1995).

For Gaussian closure we can obtain a system of 27 differential equations which consist of 6 equations for the first order moments and 21 equations for the second order moments. For non-Gaussian closure we can obtain a system of 209 differential equations which consist of 6 equations for the first order moments, 21 equations for the second order moments, 56 equations for the third order moments, and 126 equations for the fourth order moments. The systems are expressed as follows:

$$
\boldsymbol{m}' = f(\boldsymbol{m}), \quad \begin{aligned} & \boldsymbol{m} \in R^{27} \text{ for Gaussian closure} \\ & \boldsymbol{m} \in R^{209} \text{ for non-Gaussian closure,} \end{aligned} \tag{12}
$$

where $\boldsymbol{m} = \{m_{1,0,0,0,0,0}, m_{0,1,0,0,0,0}, \cdots, m_{0,0,0,0,1,1}$ or $m_{0,0,1,1,1,1}\}^T$ is the moment vector and $f(\boldsymbol{m}) = \{f_1(\boldsymbol{m}), f_2(\boldsymbol{m}), \cdots, f_{27}(\boldsymbol{m})$ or $f_{209}(\boldsymbol{m})\}^T$ is the vector field of the system.

We can obtain the equilibrium solution $\boldsymbol{m}_0$ from

$$
f(\boldsymbol{m}_0) = 0 \tag{13}
$$

In order to investigate the stability of the equilibrium solution, we let

$$
\boldsymbol{m} = \boldsymbol{m}_0 + \delta\boldsymbol{m}
$$

where $\delta\boldsymbol{m}$ is a small disturbance. The disturbance $\delta\boldsymbol{m}$ satisfies, to the first order

$$
\delta\boldsymbol{m}' = \left.\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{m}}\right|_{\boldsymbol{m}=\boldsymbol{m}_0} \delta\boldsymbol{m} \tag{14}
$$

If real parts of all eigenvalues of the Jacobian matrix are negative, the solution $\boldsymbol{m}_0$ is considered asymptotically stable.

From Eq. (13) we can see that the system (12) for Gaussian closure has the following equilibrium solution

$$
\begin{aligned}
& m_{2.0.0.0.0.0} \equiv E[X^2] = \frac{D[2\zeta_1 + 2\Psi^2\zeta_1 + \Psi(\Omega^2 + 4\zeta_1^2]}{2\Psi\Omega^2\zeta_1 P}, \\
& m_{0.0.2.0.0.0} \equiv E[X'^2] = \frac{D(2\zeta_1 + \Psi)}{2\Psi\zeta_1 P}, \\
& m_{0.0.0.0.2.0} \equiv E[U^2] = \frac{D}{\Psi\Omega^2}, \\
& m_{0.0.0.0.0.2} \equiv E[U'^2] = \frac{D}{\Psi}, \\
& m_{1.0.0.0.1.0} \equiv E[XU] = \frac{D(1 + \Psi^2 - \Omega^2 + 2\Psi\zeta_1)}{\Psi\Omega^2 P}, \\
& m_{1.0.0.0.0.1} \equiv E[XU'] = -\frac{D(\Psi + 2\zeta_1)}{\Psi P}, \\
& m_{0.0.1.0.1.0} \equiv E[X'U] = \frac{D(\Psi + 2\zeta_1)}{\Psi P} \\
& m_{0.0.1.0.0.1} \equiv E[X'U'] = -\frac{D(1 - \Omega^2)}{\Psi P}, \\
& P = 1 + \Psi^2 + \Omega^4 + 2\Psi\zeta_1(1 + \Omega^2) - 2\Omega^2(1 - 2\zeta_1^2)
\end{aligned}
\tag{15}
$$

and all other moments are zero. This equilibrium solution implies that the autoparametric vibration absorber undergoes the main system motion

(X) with no cantilever motion ($Y=0$), in other words, the motion is unimodal.

4. Monte Carlo Simulation

Monte Carlo simulation is carried out to examine the validity of the satistical closure schemes. The response statistics are estimated by numerically integrating the non-linear coupled equations (4) for a large number of sample excitation records. In the present study 400 records are found to be adequate to give numerical convergence of the results. Each record of the random excitation $W(\tau)$ with duration $\tau=4000$ is generated by sampling a sequence of 16,000 random numbers in order to prevent unacceptable frequency distortion in the record as follows (Shinozuka and Deodatis, 1991):

$$W(\tau)=\sum_{j=1}^{N}\sqrt{2(4D_j)(f_{j+1}-f_j)}\sin(2\pi\sqrt{f_jf_{j+1}}\tau+\phi_j) \quad (16)$$

where $4D_j$ are one-sided spectral density, f_j are random frequency, independent and uniformly distributed in ascending order [0, 2 Hz], and ϕ_j are random phase angles, independent and uniformly distributed on the interval [0, 2π]. The sampling time stepsize($\Delta\tau$) is chosen to be less than $1/(2f_{max})$.

5. Numerical Results

First, we solve the Jacobian matrix in equation (14) to examine the stability of the system. When the solution becomes unstable, we investigate the long-term behavior of the moments by integrating numerically the ordinary differential equation (12). Unfortunately, we can not get any solution by the non-Gaussian closure scheme because the solution experiences divergence. Figure 2 shows how the mean square values of the steady-state motion depend on the center frequency Ω of the linear filter when bandwidth Ψ is equal to 0.001 and 0.015. In Fig. 2(a) and 2(b), two curves beyond the internal resonance region($\Omega\approx 1$) predicted by the Gaussian closure scheme correspond to the equilibrium solution (15). The corresponding response is a stationary process because the mean square values are independent of τ. The results indicating that the main system motion excited directly does not encourage the cantilever motion and the responses by a stationary excitation are stationary, coincide with the response characteristics of linear systems. In the

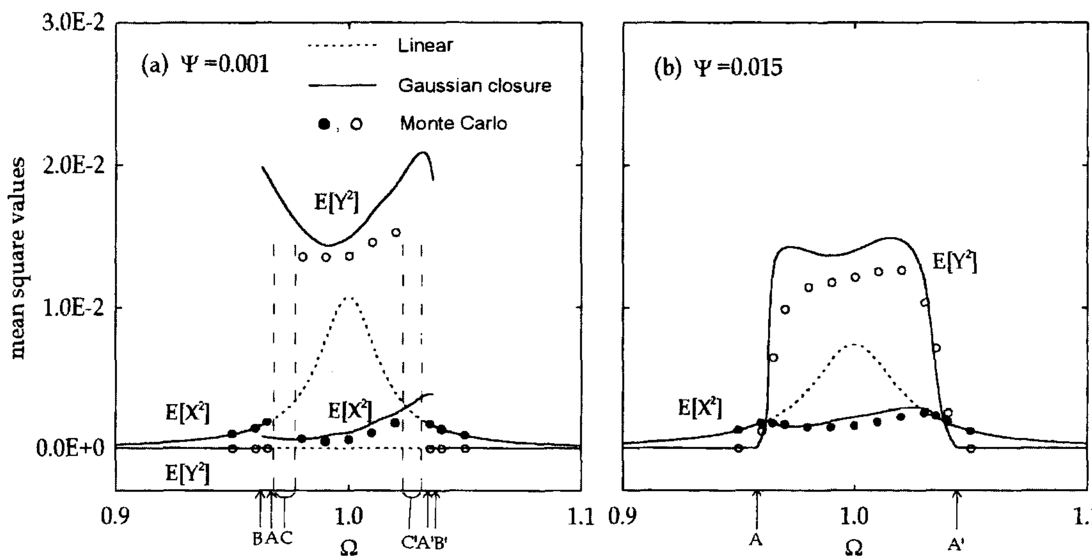

Fig. 2 Limits of mean square responses as function of center frequency Ω for $\{\zeta_1, \zeta_2, R, \varepsilon, r, E[U^2]\}=\{0.015, 0.015, 0.2, 2, 0.5, 0.00001\}$

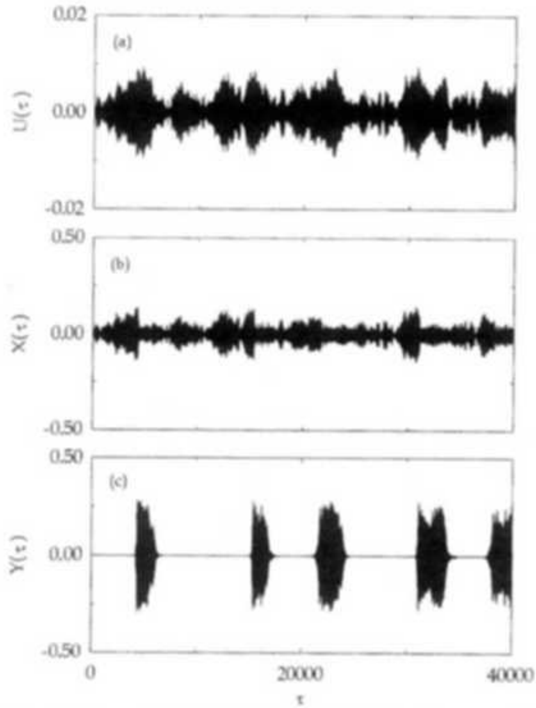

Fig. 3 Sample time history records of excitation and response according to Monte Carlo simulation for $\{\zeta_1, \zeta_2, R, \varepsilon, r, \Psi, \Omega, E[U^2]=\{0.015, 0.015, 0.2, 2, 0.5, 0.001, 0.97, 0.00001\}$

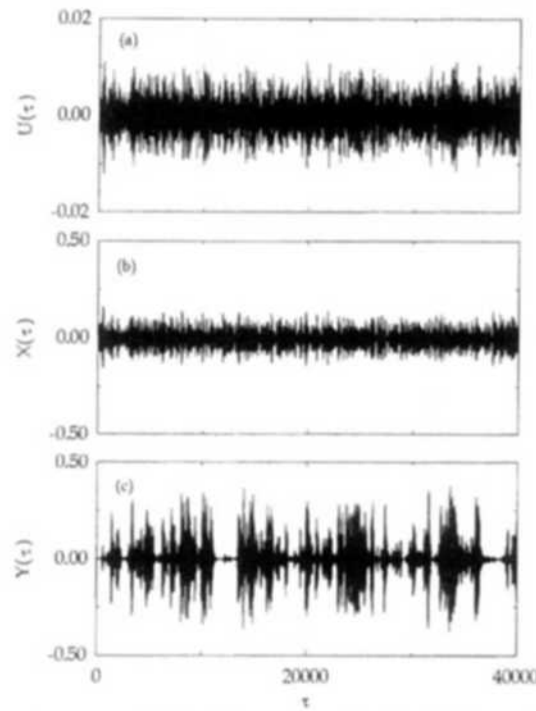

Fig. 4 Sample time history records of excitation and response according to Monte Carlo simulation for $\{\zeta_1, \zeta_2, R, \varepsilon, r, \Psi, \Omega, E[U^2]=\{0.015, 0.015, 0.2, 2, 0.5, 0.015, 0.965, 0.00001\}$

internal resonance region the results show that the energy has been transferred from the main system motion excited directly to the cantilever motion that is not excited directly.

According to the stability analysis, the equilibrium solution loses the stability at $\Omega=A$ and $\Omega=A'$ by Hopf bifurcations which occur when the Jacobian matrix of Eq. (14) has a simple pair of pure imaginary eigenvalues and no other eigenvalues with zero real parts. In addition, in Fig. 2(a) the equilibrium solution loses the stability at $\Omega=B$ and $\Omega=B'$ according to the results of numerical integration. Since $B<\Omega<A$ and $A'<\Omega<B'$ are the regions where multiple solutions exist, the system shows four jumps: upward jumps at A and A', and downward jumps at B and B'. When the center frequency Ω of the filter is increased, the upward jump at A occurs and the downward jump at B' occurs. On the contrary, as the center frequency Ω is decreased the upward jump at A' occurs and the downward jump at B occurs.

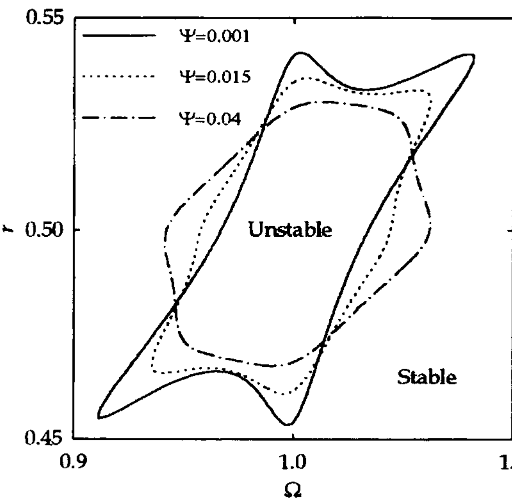

Fig. 5 Stability boundaries predicted by Gaussian closure according to Ψ in $r-\Omega$ plane for $\{\zeta_1, \zeta_2, R, \varepsilon, E[U^2]\}=\{0.015, 0.015, 0.2, 2, 0.00001\}$

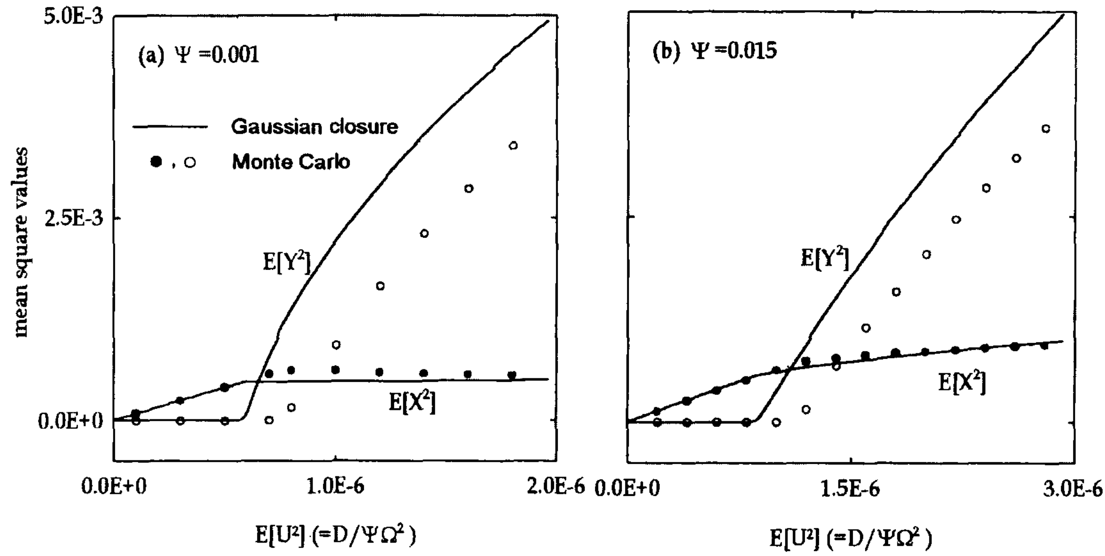

Fig. 6 Limits of mean square responses as function of $E[U^2]$ for $\{\zeta_1, \zeta_2, R, \varepsilon, r, \Omega\}=\{0.015, 0.015, 0.2, 2, 0.5, 0.99\}$

As shown in the figures it can be seen that responses estimated by Monte Carlo simulation are in good agreement with those predicted by Gaussian closure. In Fig. 2(a), jumps estimated by Monte Carlo simulation exist at narrow ranges (C and C′) as the cantilever motion $Y(\tau)$ shown in Fig. 3. On the other hand, Fig. 2(b) reveals the disappearance of jump phenomenon as shown in Fig. 4. Figure 5 shows the stability boundaries predicted by the Gaussian closure scheme according to Ψ in $\Omega-r$ planes.

Figure 6 presents limits of mean square displacements as functions of the mean square excitation $E[U^2](=D/\Psi\Omega^2)$ when bandwidth Ψ is equal to 0.001 and 0.015. For the region of mean square excitation below the Hopf bifurcation point, the mean square values of the main system motion excited directly increase linearly as $E[U^2]$, while the mean square values of the cantilever motion remain at zero. In other words, the system response shows the response characteristics of a linear system when $E[U^2]$ exists below the Hopf bifurcation point. For the region of mean square excitation above the Hopf bifurcation point, the limits of mean square values of the main system motion is saturated when $\Psi=0.001$. On the other hand, the limits of mean square values of the cantilever motion increase as $E[U^2]$. Thus there exists saturation, which implies a phenomenon in which the motion excited directly stops increasing when the excitation level reaches a critical value. But as the bandwidth increases from $\Psi=0.001$ to $\Psi=0.015$, the saturation phenomenon disappears. The Gaussian closure predicts the bifurcation of the cantilever motion prior to that point predicted by the Monte Carlo simulation.

6. Conclusions

In order to investigate the influences of the internal resonance on the system responses of a two-degree-of-freedom system with a narrow band random excitation, we examined an autoparametric vibration absorber with a narrow band random excitation to the main mass. The narrow band random process is generated from a linear shaping filter excited by a white noise. For sufficiently small bandwidth of the linear shaping filter, the jump phenomenon of the cantilever mode and the saturation phenomenon of the main system are predicted by both the analysis and numerical simulation. The saturation phenomenon occurs over a finite range of the internal detuning parameter. But as the bandwidth increases the jump and saturation phenomena disappear. The Gaussian closure solution predicts the bifurcation of the cantilever motion to occur at relatively lower excitation level than the Monte Carlo simulation.

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